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Optimising Throughput for Planning Building Evacuations

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Abstract

An M/G/C/C state dependent queuing network can be used to explicitly represent the complicated behaviour of pedestrians in a restricted space layout. The method to investigate pedestrian movements along different corridors in a topological network is implemented in this research. The M/G/C/C analytical model was used to quantify the optimal arrival rates and their effects on the corridors' performance in terms of throughput, blocking probability, expected number of occupants in the system, and estimated travel time. The optimal arrival rates were then determined by feeding these optimum arrival rates into its Network Flow Programming model.

Keywords: M/G/C/C state dependent; Topological network; Queuing system; Network flow model; Performance assessment

Introduction

Systems that have several queues connected by things (like clients and items) that are vying for services are known as queuing systems. Service (bank, cafeteria, and healthcare), telecommunication (data packet, contact centre, and computer network), transportation (airport and seaport), and manufacturing (production line) systems all frequently face these situations. Regardless of the amount of residing entities, the service times in these systems vary based on a statistical distribution. Depending on the current number of entities, other systems dynamically modify their service times. The scenarios are common for things moving through a layout with restricted space, such as cars on a road and pedestrians crossing a corridor. A state-dependent queuing network that is M/G/C/C can be used to model the behaviour of the entities 1-3. This paper is organized as follows. We first discuss the mathematical background governing the *M/G/C/C* approach and how it can be used to measure pedestrian traffic and congestion. We then briefly describe the structure of the hall used in our case study and present the best arrival of each available corridor maximizing its throughput. We next discuss the strategy to achieve the best performance of the hall to efficiently flow its occupants. Finally, some concluding remarks are given in the last section. Markovian arrival processes are denoted by M in this Kendall notation, general state dependent service rates by G, parallel servers by C, and the overall capacity of a circulation space by C.

In this research, pedestrian flows in a complicated hall with series, merging, and splitting networks are measured using the M/G/C/C technique. Our goal is to determine the most effective way to evacuate pedestrians from the hallway while adhering to the present Standard

Operating Procedures (SOP) in the event of an emergency. We started by creating an M/G/C/C analytical model especially for the hall in order to do this. We then used the model to determine the optimal arrival rate for each available corridor in order to maximise its throughput. We then utilised the network flow programming approach 4 to determine the optimal arrival rates to source corridors, maximising the hall's overall throughput, while taking into account the best arrival rate and the entire network of corridors. We next used Arena software 8-10 to create a discrete event simulation (DES) model 5-7 in order to verify the analytical findings. In particular, the M/G/C/C state dependent queuing network can be modelled using this commercial program, as explained in detail in 11.

Methodology

The M/G/C/C strategy uses a building's or environment's circulation area as a server for the entities that make requests. The number of servers is the same as the space's capacity. When a space is filled, pedestrians are not permitted to take it; instead, they must wait in queue to be served. Furthermore, the current service time—that is, the walking speed of the pedestrians—is determined by the present number of pedestrians.

The effect of the number of pedestrians to the current walking speed was formulized by Yuhaski and Smith [3]. They presented linear and exponential models of walking speed as follows:

Linear: $$V_n = \frac{V_1}{c}(c+1-n) \tag{1}$$

Exponential: $$V_n = A\exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right] \tag{2}$$

where
γ, β = Shape and scale parameters for the exponential model,
V_n = Average walking speed for n pedestrians in a corridor,
V_a = Average walking speed when crowd density is 2 peds/m^2 = 0.64 m/s,
V_b = Average walking speed when crowd density is 4 peds/m^2 = 0.25 m/s,
V_1 = Average walking speed for a single pedestrian = 1.5 m/s,
n = Number of pedestrians in a corridor,
$a = 2 \times l \times w$,
$b = 4 \times l \times w$,
$c = 5 \times l \times w$,
l = corridor length in meters, and
w = corridor width in meters.

Based on the models, Yuhaski and Smith [3] developed the limiting probabilities for the number of pedestrians in an *M/G/C/C* model as follows:

$$P_n = \frac{[\lambda E(S)]^n}{n!f(n)f(n-1)\ldots f(2)f(1)}P_0 \quad n = 1, 2, 3, \ldots, c \tag{3}$$

where, $$P_0^{-1} = 1 + \sum_{n=1}^{C}\left[\frac{[\lambda E(S)]^i}{i!f(i)f(i-1)\ldots f(2)f(1)}\right].$$

In this model, λ is the arrival rate to a corridor, $E(S)$ is the expected service time of a single pedestrian in the corridor, P_n is the probability when there are n pedestrians in the corridor, P_0 is the probability when there is no pedestrian in the corridor, and $f(n)$ is the service rate and is given by $f(n) = \frac{V_n}{V_1}$. c meanwhile refers to the capacity of the corridor. Any pedestrians attempting to enter the full capacity corridor will be blocked. The probability of such blocking

(P_{balk}) is equal to P_n where n equals to c. Since Cheah and Smith [12] showed that *M/G/C/C* networks are equal to *M/M/C/C* networks, various performance measures of the corridor can then be computed as:

$$\theta = \lambda(1 - P_{balk}), \quad E(N) = \sum_{n=1}^{C} nP_n \quad \text{and} \quad E(T) = \frac{E(N)}{\theta} \tag{4}$$

where θ is the throughput of the corridor (in pedestrians per second; i.e., peds/s), *E(N)* is the expected number of pedestrians in the corridor and *E(T)* is the expected service time in seconds.

USAS's Network as a Case Study

We take into consideration the USAS (Universiti Sultan Azlan Shah) hall as a setting for putting the M/G/C/C method into practice. Figure 1 displays the hall's graphical layout. The letters A through I stand for the corridor entrance doors, the numbers for the corridors, and the letters A', B', and C' for the exits to open areas.

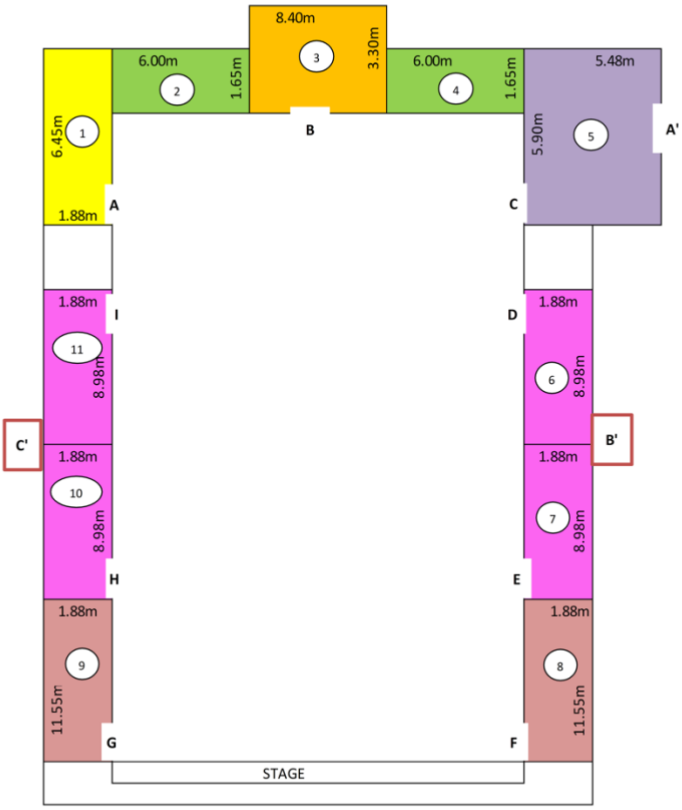

Figure 1: USAS's Hall Structure

Table 1 represents the dimensions of the corridors in terms of their lengths, widths (at entrances and exits) and capacities. A larger capacity supports more occupants. Corridor *C'* has the biggest space, followed by corridors 3, 8, 9 and 5.

Table 1: Corridor Dimensions and Capacities

Corridor	**Length (m)**	**Width**		**Capacity (occupant)**
		Entrance (m)	**Exit (m)**	
1	6.45	1.88	-	61
2	6.00	1.65	-	50
3	8.40	3.30	-	139
4	6.00	1.65	-	50
5	5.48	1.77	5.90	105
6	8.98	1.88	-	84
7	8.98	1.88	-	84
8	11.55	1.88	-	109
9	11.55	1.88	-	109
10	8.98	1.88	-	84
11	8.98	1.88	-	84
B’	3.60	4.00	-	72
C’	10.00	3.00	-	150

The hall has thirteen corridors. Source corridors are 1, 3, 5, 6, 7, 8, 9, 10, and 11. The intermediary corridors are 2 and 4. In order to leave the hall by doors A, B, C, D, E, F, G, H, and I, occupants select the source corridors that are closest to them. They then take exits A', B', and C' to reach the nearby open spaces. Door A, B, and C occupants select exit A'. While those occupying doors I, H, and G select exit C', those occupying doors D, E, and F select exit B'. While exit C' is a staircase leading to the open regions, exits A' and B' are the exit corridors. Table 2 shows the correlation between the corridors' arrival rates () and throughputs () when the facility is viewed as a network. The optimal arrival rates and their effects on throughputs, blocking probability (p(c)), estimated number of occupants (L), and average trip time (W) are also displayed in the table.

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Table 2: Arrival Rates and Throughputs of Corridors

Corridor		λ	Total λ	θ	Best λ	Best θ	p(c)	L	W
Source Corridors	1	λ_1	λ_1	θ_1	1.99718	1.95116	0.02304	19.60526	10.04801
	3	λ_3	$\lambda_3 = \lambda_3 + \theta_2$	θ_3	3.55649	3.52221	0.00964	37.26319	10.57950
	5	λ_5	$\lambda_5 = \lambda_5 + \theta_4$	θ_5	4.12374	4.07036	0.01294	29.74330	7.30729
	6	λ_6	λ_6	θ_6	2.01882	1.98558	0.01647	25.26585	12.72465
	7	λ_7	$\lambda_7 = \lambda_7 + \theta_8$	θ_7	2.01882	1.98558	0.01647	25.26585	12.72465
	8	λ_8	λ_8	θ_8	2.01792	1.99291	0.01240	30.33084	15.21941
	9	λ_9	λ_9	θ_9	2.01792	1.99291	0.01240	30.33084	15.21941
	10	λ_{10}	$\lambda_{10} = \lambda_{10} + \theta_9$	θ_{10}	2.01882	1.98558	0.01647	25.26585	12.72465
	11	λ_{11}	λ_{11}	θ_{11}	2.01882	1.98558	0.01647	25.26585	12.72465
Intermediate Corridor	2	λ_2	$\lambda_2 = \theta_1$	θ_2	1.76335	1.71053	0.02995	17.61650	10.29886
	4	λ_4	$\lambda_4 = \theta_3$	θ_4	1.76335	1.71053	0.02995	17.61650	10.29886

Exiting to Open Spaces	A'	$\lambda_{A'}$	$\lambda_{A'} = \theta_5$	$\theta_{A'}$	-	-	-	-	-
	B'	$\lambda_{B'}$	$\lambda_{B'} = \theta_6 + \theta_7$	$\theta_{B'}$	4.30450	4.21867	0.01994	22.84601	5.41545
	C'	$\lambda_{C'}$	$\lambda_{C'} = \theta_{10} + \theta_{11}$	$\theta_{C'}$	3.25133	3.22194	0.00904	40.39662	12.53799

We then employed the network flow programming technique to determine the optimal source corridor arrival rates and routing plans that maximised the hall's overall throughput using the data in Table 2. As long as the flow out of each corridor equals the flow in, and each corridor's maximum arrival rate is less than or equal to its best arrival rate, the goal function is to maximise the overall flow out of the exit corridors.
with heavier workloads having a negative effect and lighter workloads having a positive effect.

Result and Discussion

Table 3 displays the source corridors' performance metrics for arrival rates that maximise source corridor throughputs. Its simulation model was then used to validate these analytical findings. Arena 14.0's Process Analyser was used to perform the simulation model. The provided simulation results were all based on 30 replications, each running for 20,000 seconds.

Table 3: Analytical and Simulation Arrival Rates Maximizing the Total Throughput of the Hall

Model	Corridor	λ	L	W	p(c)	θ
Analytic	1	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	2	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	3	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	4	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	5	4.1237	29.7403	7.3065	0.0129	4.0704
Simulation			39.9528	11.0777	0.0494	3.9158
Analytic	6	2.0188	25.2627	12.7231	0.0165	1.9856
Simulation			44.2097	25.9429	0.1013	1.8100
Analytic	7	2.0188	25.2627	12.7231	0.0165	1.9856
Simulation			44.2097	25.9429	0.1013	1.8100
Analytic	8	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	9	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	10	2.0188	25.2627	12.7231	0.0165	1.9856
Simulation			44.2097	25.9429	0.1013	1.8100
Analytic	11	1.2325	9.3071	7.5514	0.0000	1.2325
Simulation			9.3031	7.5534	0.0000	1.2316
Analytic	*B'*	3.9712	15.4011	3.8792	0.0003	3.9702
Simulation			22.8379	6.2682	0.0295	3.8537
Analytic	*C'*	3.2181	37.0272	11.5391	0.0029	3.2088
Simulation			35.7511	11.1171	0.0000	3.2158
Analytic	**Total throughput of the network**					**11.2493**
Simulation						**10.9853**
Difference (%)						2.3468

In order to arrange for evacuation, we should move the residents straight to corridor 5 at a rate of 4.1237 people per second, leaving through corridor A'. If an occupant is close to exit B, they should proceed to corridors 6 and 7 at a speed of 2.0188 ped/s. Since residents must contend with those from corridors 6 and 7, we should prevent them from entering corridor 8 through door F. This also applies to corridor 9. Only corridors 10 and 11 should be used by occupants who are close to exit C' in order to leave for open spaces, which have corresponding speeds of 2.0188 and 1.2325 ped/s. The hall's maximum throughput is 11.2493 ped/s. The throughput disparity between the simulation and analytical results was only somewhat smaller, as can be seen.

Conclusion

This study demonstrates that inhabitants of a complicated topological network can be effectively evacuated using an M/G/C/C state dependent queuing network. We selected the USAS hall as a case study and determined the optimal arrival rates to maximise the throughput of its accessible source corridors. Arrival rates that are higher than the recommended levels will result in congestion rather than increased throughput. Pedestrian traffic flows smoothly at any lower arrival rates than the values, although throughput will soon suffer. To guarantee the optimal throughput, it is crucial to properly regulate the arrival rates and direct pedestrians as they move from the source to the departure pathways.

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